NORTH SYDNEY GIRLS HIGH SCHOOL

HSC MATHEMATICS ASSESSMENT TASK

TERM 2 – 2006

Time Allowed: 1 hour + 2 minutes reading time

Instructions:

- Start each question on a new page
- Write on one side of the paper only, work down the page and do not work in columns
- Leave a margin on the left hand side of the page
- Show all necessary working
- Marks may not be awarded for untidy or poorly arranged work
- Diagrams are not drawn to scale
- There are five questions
- Marks are as indicated

This task is worth 20% of the HSC Assessment Mark

Name:______________________________________

Question 1 (7 marks) **Marks**

a. Evaluate $\log_8 32$ **1**

b. Simplify $\log_5 45 + \log_5 40 - \log_5 72$ **1**

c. Given that $\log_2 3 = 1{\cdot}58$, find $\log_2 12$ **1**

d. Find $\int_0^1 \left(e^x + 1\right) dx$ in exact form **2**

e. Evaluate $\int_0^3 \frac{x}{1+x^2} dx$ **2**

Question 2 (10 marks) **Start a new page.**

a. For the curve $y = 3\sin 2x$

i. State the period and amplitude. **2**

ii. Sketch the curve between $0 \le x \le 2\pi$ **1**

b. Find all solutions of $2\sin x + 1 = 0$ in the interval $0 \le x \le 2\pi$.
(Give your answer/s as exact values). **2**

c. Differentiate the function $y = 3\sin x + 4\cos x$ **2**

d. Evaluate $\int_0^{\frac{\pi}{4}} \left(\sin 2x + \sec^2 x\right) dx$ **3**

Question 3 (9 marks) **Start a new page.**

a. The following table gives the values of $f(x) = x\log x$. Use Simpson's Rule with these 5 function values to find an approximate value of $\int_1^5 x\log x\, dx$.
(Give your answer to 2 decimal places) **3**

x	1	2	3	4	5
$f(x)$	0	1·39	3·3	5·55	8·05

b.

i. Find the derivative of tan2*x*. **1**

ii. Find the volume of the solid formed when the curve $y = \sec 2x$ is rotated about the *x*-axis between the ordinates $x = 0$ and $x = \frac{\pi}{8}$. **2**

c.

i. The *x*-coordinate of one point of intersection between the line $y = x + 1$ and the parabola $y = 2x^2$ is $x = -\frac{1}{2}$. Using quadratic equations, or otherwise, find the *x*-coordinate of the other point of intersection. **1**

ii. Hence, find the area enclosed between the line and the parabola. **2**

Question 4 (9 marks) **Start a new page.** **Marks**

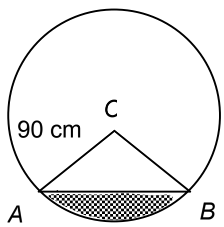
a. The area under the curve $y = \dfrac{1}{x}$ between $x = 1$ and $x = b$ is equal to 1 square unit. What is the value of b? **3**

b. A solid of revolution is formed by rotating about the x-axis the curve $y = 2(1 + e^{2x})$ between the ordinates $x = 0$ and $x = \dfrac{1}{2}$. Show the volume of the solid is $\pi(e^2 + 4e - 3)$ units^3. **3**

c. For the curve $y = \log(1 + \sin x)$, show that the second derivative is given by $\dfrac{-1}{1 + \sin x}$ **3**

Question 5 (8 marks) **Start a new page.**

a. In the diagram the length of the arc AB is 50 cm. The radius of the circle is 90 cm.



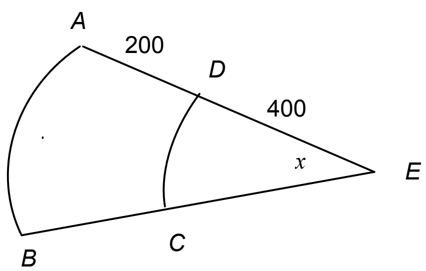
i. Find $\angle AOB$ in radians . **1**

ii. Find the shaded area correct to the nearest cm^2. **2**

b. In the figure AB and CD are circular arcs which subtend an angle x radians at the centre E, where $0 \le x \le \pi$. The length AD is 200 metres and DE is 400 metres.

i. Find the length of each of the arcs AB and CD in terms of x. **2**

ii. A person lives at A and wants to walk to the train station at B. The paths AB, BC, CD and DA form a road. For what values of x is it shorter for the person to walk along the route $ADCB$ rather than along the arc AB? **3**



END OF TEST

Solutions

(1) (a) $\log_8 32 = x$

$\therefore 8^x = 32 = 2^5 \Rightarrow 2^{3x} = 2^5$

$\therefore x = \dfrac{5}{3}$

Alternatively

$\log_8 32 = \dfrac{\ln 32}{\ln 8} = \dfrac{5}{3}$ (calculator)

(b) $\log_5 45 + \log_5 40 - \log_5 72 = \log_5\left(\dfrac{45 \times 40}{72}\right) = \log_5 25 = 2$

(c) $\log_2 12 = \log_2\left(2^2 \times 3\right) = \log_2\left(2^2\right) + \log_2(3) = 2 + \log_2(3) \approx 3{\cdot}58$

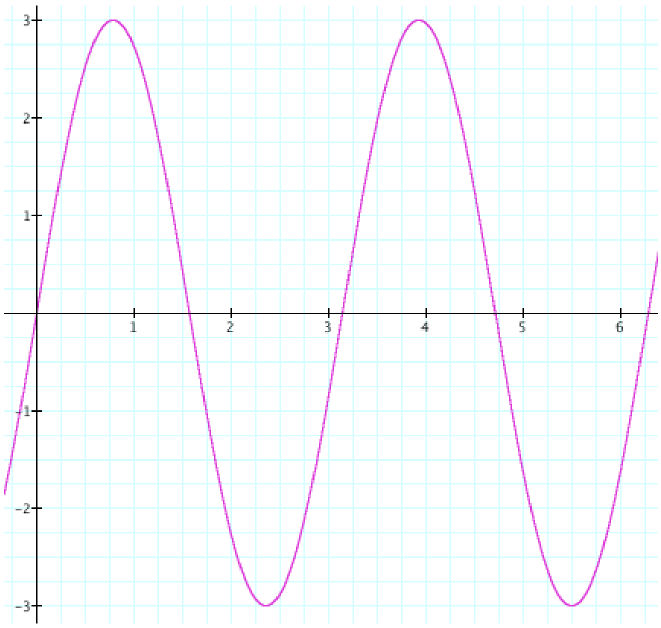
(d) $\int_0^1 \left(e^x + 1\right)dx = \left[e^x + x\right]_0^1 = (e+1) - (1+0) = e$

(e) $\int_0^3 \dfrac{x}{1+x^2}\,dx = \dfrac{1}{2}\int_0^3 \dfrac{2x}{1+x^2}\,dx = \dfrac{1}{2}\left[\ln\left(1+x^2\right)\right]_0^3 = \dfrac{1}{2}(\ln 10 - \ln 1) = \dfrac{1}{2}\ln 10$

(2) (a) (i) Amplitude = 3

Period $= \dfrac{2\pi}{2} = \pi$

(ii)



(b) $2\sin x + 1 = 0 \Rightarrow \sin x = -\dfrac{1}{2}$

$\therefore x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

(c) $y = 3\sin x + 4\cos x$

$\therefore y' = 3\cos x - 4\sin x$

(d) $\int_0^{\frac{\pi}{4}} \left(\sin 2x + \sec^2 x\right)dx = \left[-\dfrac{\cos 2x}{2} + \tan x\right]_0^{\frac{\pi}{4}} = (1) - \left(-\dfrac{1}{2}\right) = \dfrac{3}{2}$

(3) (a)

x	1	2	3	4	5
$f(x)$	0	1·39	3·3	5·55	8·05
w	1	4	2	4	1
$w \times f(x)$	0	5·56	6·6	22·2	8·05

$h = 1$

$$\sum wf(x) = 42 \cdot 41 \Rightarrow \int_1^5 x \log x dx \approx \frac{h}{3} \times 42 \cdot 41 = 14 \cdot 14 \text{ (2dp)}$$

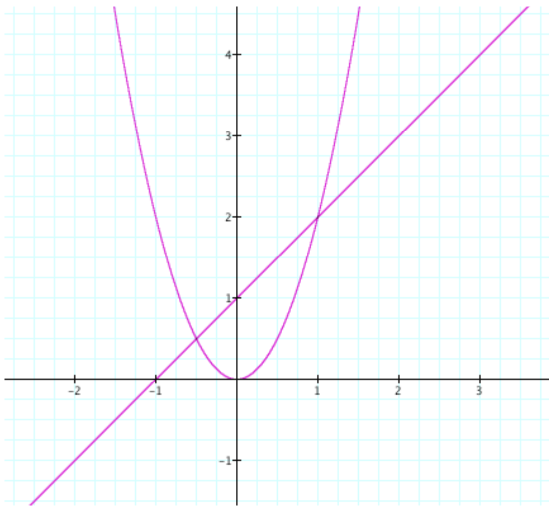
(b) (i) $\frac{d}{dx}(\tan 2x) = 2\sec^2 2x$

(ii) $V = \pi \int_0^{\frac{\pi}{8}} \sec^2 2x dx = \frac{\pi}{2} \int_0^{\frac{\pi}{8}} 2\sec^2 2x dx$

$= \frac{\pi}{2}[\tan 2x]_0^{\frac{\pi}{8}} = \frac{\pi}{2} \times 1$

$= \frac{\pi}{2}$ cu

(c) (i)



Let $\alpha = -\frac{1}{2}$ and β be the x-coordinates of the points of intersection

$y = x + 1 = 2x^2 \Rightarrow 2x^2 - x - 1 = 0$

$\therefore \alpha + \beta = \frac{1}{2} \Rightarrow -\frac{1}{2} + \beta = \frac{1}{2} \Rightarrow \beta = 1$

(ii) Area $= \int_{-\frac{1}{2}}^{1} (x + 1 - 2x^2) dx = \left[\frac{x^2}{2} + x - \frac{2}{3}x^3\right]_{-\frac{1}{2}}^{1} = 1\frac{1}{8}$

(4) (a) $\int_1^b \frac{dx}{x} = 1 \Rightarrow [\ln x]_1^b = 1$

$\therefore \ln b = 1 \Rightarrow b = e$

(b) $V = \pi \int_0^{\frac{1}{2}} 4\left(1+e^{2x}\right)^2 dx = \pi \int_0^{\frac{1}{2}} \left(4+8e^{2x}+4e^{4x}\right) dx$

$= \pi\left[4x + 4e^{2x} + e^{4x}\right]_0^{\frac{1}{2}}$

$= \pi\left[2 + 4e + e^2 - (4+1)\right]$

$= \pi\left[e^2 + 4e - 3\right]$ cu

(c) $y = \ln(1+\sin x)$

$y' = \frac{\cos x}{1+\sin x}$

$y'' = \frac{(1+\sin x)\times(-\sin x) - \cos x \times \cos x}{(1+\sin x)^2}$

$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \frac{-\sin x - 1}{(1+\sin x)^2}$

$= -\frac{1+\sin x}{(1+\sin x)^2}$

$= -\frac{1}{1+\sin x}$

(5) (a) (i) $90 \times \angle AOB = 50$

$\therefore \angle AOB = \frac{5}{9}$

(ii) Let $\theta = \angle AOB$

Shaded area $= \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}\times 90^2\left(\frac{5}{9} - \sin\frac{5}{9}\right)$

$\approx 114 \text{ cm}^2$

(b) (i) $CD = 400x$; $AB = 600x$

(ii) $ADCB = 400 + 400x$

$ADCB <$ arc $AB \Rightarrow 600x > 400 + 400x$

$\therefore 200x > 400$

$\therefore x > 2$

So when the angle is greater than 2 radians (ie greater than $114°35'$), arc AB is the greater distance